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The effectiveness of using manipulatives to teach fractions

Jaime Gaetano

THE EFFECTIVENESS OF USING MANIPULATIVES TO TEACH FRACTIONS

by
Jaime Gaetano

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Thesis Chair: Roberta Dihoff, Ph.D.

Abstract

Jaime Gaetano
The Effectiveness of Using Manipulatives to Teach Fractions
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Roberta Dihoff, Ph.D.
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The current study will focus on the effectiveness of using manipulatives when teaching fractions to elementary school students. Learning the concepts of fractions can be one of the most difficult skills to master for elementary level students. With so many different ways to expose students to manipulatives and enhance their fraction learning experience, it is important to examine how effective these teaching tools can be with respect to student achievement.

The current study will discuss the effectiveness on student achievement when manipulatives are used during the teaching process. The main focus will be on student growth after being taught concepts of fractions including addition and subtraction while using manipulatives to engage them in their lessons. The students involved in this study are in one fourth grade class. This class includes 18 students that are performing at various achievement levels. Some of the participants have specific learning disabilities which hinder their ability to retain mathematical concepts without repetition over a longer period of time. The lessons being taught are included in the Everyday Mathematics fourth grade curriculum for fraction concepts. This curriculum is the Vineland Public Schools district wide mathematics curriculum. The teachers are responsible for teaching this curriculum using manipulatives for specific lessons.

The study is taking place of a time span of four weeks. They will be tested prior to being taught the unit on fractions. They will be divided into two groups: one group will be instructed using integration of manipulatives and the other group will be instructed using worksheets and direct instruction including teacher modeling. Both groups will be given a post test to determine if the use of manipulatives was effective. This study will consist of comparing students' assessment scores when being taught the concepts of fractions while using manipulatives and students' assessment scores when they are taught without the use of manipulatives. An independent sample T-test revealed that students working with manipulatives during instructional time, small group time, and independent tasks demonstrated a significant amount of growth as compared to their peers that did no use manipulatives during any time of the learning process.

Table of Contents

List of Figures

Chapter 1

Introduction

The current study will focus on the effectiveness of using manipulatives when teaching fractions to elementary school students. McBride and Lamb (1986) explain that educators have different ways of implementing manipulatives as teaching tools in the classroom. Belenky and Nokes (2009) discuss that some educators believe that students learn concepts and problem solve better when using hands-on, concrete models. Others feel manipulatives are a way of allowing the students additional play time in the classroom and are not thought of as teaching tools according to Moyer (2001).

Furthermore, many teachers do not use manipulatives as teaching tools because they have difficulty finding the time in their daily teaching schedule (Joyner, 1990). Learning the concepts of fractions can be one of the most difficult skills to master for elementary level students. With so many different ways to expose students to manipulatives and enhance their fraction learning experience, it is important to examine how effective these teaching tools can be with respect to student achievement.

The current study will discuss the effectiveness on student success when manipulatives are used during the teaching process. The main focus will be on student growth after being taught concepts of fractions including addition and subtraction while using manipulatives to engage them in their lessons. I propose that if educators use manipulatives when teaching fractions, then students would successfully internalize fractional concepts thus demonstrating significant student growth. I propose the use of manipulatives when teaching fractions is more effective than teaching fractions using the paper pencil style of teaching the concepts.

Manipulatives as defined by Moyer (2001) as “physical objects designed to represent explicitly and concretely mathematical ideas that are abstract.” Examples include: commercial objects, such as Algeblocks, pattern blocks, or virtual/ computer-based manipulatives (Jones, Uribe-Florez & Wilkins, 2011). McNeil and Jarvin (2007) define manipulatives as any object that can be used to help students understand the concepts of mathematics even if they were not directly intended for that specific purpose. Examples include: folding paper, tiles, pie pieces, geoboards, teacher-made pictures, etc. For this study, we are using all of the above as appropriate definitions of manipulatives.

Intrinsic motivation is defined by Ryan and Deci (2000) as “the inherent tendency to seek out novelty and challenges, to extend and exercise one’s capacities, to explore, and to learn.” Schunk, Pintrich and Meece (2008) state students that are interested in an activity are more likely to be motivated to choose and persist at the activity. Researchers call this intrinsic motivation and believe that students will internalize the concepts learned from the enjoyed activity better than that of one they have no interest in participating (Ryan & Deci, 2000, pg. 70).

The students involved in this study are in one fourth grade class. This class includes eighteen students that are performing at various achievement levels. The students are a mixture of fourteen girls and four boys from various ethnic backgrounds. The lessons being taught are included in the Everyday Mathematics fourth grade curriculum for fractional concepts. This curriculum is the Vineland Public Schools district wide mathematics curriculum. The teachers are responsible for teaching this curriculum using manipulatives for specific lessons.

The study is taking place of a time span of three weeks which is not a significant amount of time. In addition, some of the participants have specific learning disabilities which hinder their ability to retain mathematical concepts without repetition over a longer period of time. An additional limitation with this study is that the assessments are paper-pencil based and do not include oral responses or virtual assessments which may be necessary for some students to successfully demonstrate their mastery of the skill. Another limitation is that this study involves only one class in-class resource class resulting in having only 18 participants.

In summary, the current literature review focused on the effectiveness of teaching fractions with the use of manipulatives as both teaching tools and learning enhancements. The importance of methodology when teaching with these manipulatives is discussed as well.

This study will consist of student assessment scores when being taught the concepts of fractions while using manipulatives. The class was divided into two groups including a mix of boys and girls at various academic ability levels based on SuccessMaker scores. I administered a pre assessment to both groups. One group was taught a series of concepts involving fractions while having them engaged using manipulatives during both teaching of lessons and independent work. The other group was taught the same concepts using only teacher models and worksheets. This process will occur over a period of four weeks during the time period allotted from the Vineland Public Schools Mathematics pacing guide. Both groups were administered a post assessment and the results were analyzed based on student growth.

Chapter 2

Literature Review

In the past, mathematics has been taught procedurally by remembering specific steps that would bring the student to the correct answer (McLeod, Vasinda & Dondlinger, 2012). Current research is indicating that mathematics is more than simply learning procedures by rote memorization and writing answers on worksheets. It is about understanding rules through mathematical thinking and abstract reasoning. According to Moyer (2001) students must understand what they are learning on order for it to be permanent. Belenky & Nokes (2009) discuss how using concrete materials along with metacognitive prompts by teachers is critical to internalizing complex cognitive problem solving skills. The use of manipulatives grounds new information in prior knowledge and enables students to abstract the critical features through reflection thus leading to higher student achievement.

Types of Manipulatives

Manipulatives are materials that are used to assist students' mathematical learning in more meaningful ways (Stein & Bovalino, 2001). These concrete materials assist children at all levels of education including understanding processes, communicating their mathematical thinking, and extending their ideas to higher order thinking levels (Balka, 1993). Using manipulatives enables students to make connections with other mathematical topics, gain insight to other academic subject areas, and in their personal interests and experiences (Lee & Chen, 2010). "Students' mental images and abstract ideas are based on their experiences. Hence, students who see and manipulate a variety of objects have clearer mental images and can represent abstract ideas more completely than those whose experiences are meager" (Kennedy, 1986, p. 6).

There is a long history of manipulative use when teaching mathematics. Johann Pestalozzi (1746-1827) influenced educators in the 19th century to teach young children number sense through the use of manipulatives including basic blocks (Saettler, 1990). As Maria Montessori was teaching young children in the first Montessori school in 1907, she quickly learned that children learn best when they are free to explore using hands-on materials such as beads, puzzles, and wooden shapes (Encyclopedia of Social Reforms, 2013). Piaget's constructivism perspective of the 1970s, states that conceptual knowledge is founded through discovering while using objects rather than through hearing information via person to person (Piaget, 1973). Today, there are many different types of manipulatives ranging from virtual computer software programs to teacher-made materials.

Manipulatives are concrete materials that range in size, shape, and color. They include but are not limited to physical models such as fraction circles, Cuisenaire rods, paper folding, pie pieces, fraction tiles, dice, and chips that allow students to develop mental images for fractions (Ball, 1992, Cramer & Henry, 2013). Manipulatives are not rulers, projectors, or calculators. However, computers are included as a manipulative because they simulate such concrete materials (Johnson, 1993). Researchers Martin and Schwartz (2005) discuss how children learn fraction concepts better using pie pieces rather than tiles. Pie pieces better embody the fraction concept and better illustrate the part verses the whole concept. Boggan, Harper and Whitmire (2010) suggest using fractions strips to add and subtract fractions or to represent equivalent fractions. McBride and Lamb (1989) illustrate an easy, inexpensive way to create concrete materials by simply duplicating circular pieces using tagboard and cutting them into sets including 2 wholes, 4 halves, 8 fourths, 16 eighths, 6 thirds, and 12 sixths. This helps students concretely retain the concept of part and whole. Hiebert (1997) states tools are unavoidable and essential when learning mathematical concepts.

Virtual manipulatives are the most cost effective and timely manipulative tool in the classroom. Virtual manipulatives are interactive, web-based visual representations that allow students to imitate using concrete materials (Moyer, 2002). Several websites have been developed to give teachers free access to use with their students (Bouck & Flanagan, 2009). These websites can be used to reinforce instruction practice as well as expand the boundaries of assessment (Johnson, Campet, Gaber & Zuidema, 2012). However, the computer program must not be the only source of instruction. The National Council of Teachers of Mathematics' Principles and Standards for School Mathematics states that students should learn mathematics through creating and using representations by organizing, recording, and communicating ideas. They should be selecting, applying, and translating among representations to solve problems. They should be using materials to model and interpret physical, social, mathematical phenomena (NCTM, 2000).

Implementation and Effectiveness

Teachers must realize that students must be able to visualize concepts beyond the experience of using the computer (Moyer-Packenham, Ulmer & Anderson, 2012). Suh, Moyer and Heo (2005) discuss a positive to virtual manipulative use is that many have teacher prompts already embedded into the activities which allows for students to make sense of mathematical concepts. This would allow for teachers to use their time preparing the concrete materials used in this connected learning experience. Other positive factors are that they are easy to manage in the classroom, are available if students have computer access, and older students feel computers are more age appropriate than hands-on manipulatives according to Moyer, Bolyard and Spikell (2002). Clements and McMillen (1996) summarize a number of advantages to using virtual manipulatives including: student motivation and focus attention, flexibility, retrieval of student progress, and assessment.

When using virtual manipulatives as an assessment tool, teachers should consider guidelines prior to selecting them according to Johnson, Campert, Gaber and Zuidema (2012). Consideration of the extent to which it addresses the target concept, the way in which it takes advantage of technology, and how elicit responses would give concrete insight into the students' learning need to be addressed. In addition to individual assessment, small group work can also be utilized on the computer. Virtual manipulatives also increase peer interactive learning groups. Clements (2002) discusses how children prefer to work together than alone. They have the ability to use the keyboard so they begin the activity with a sense of pride thus displaying more positive emotion and interest in the learning activity. This allows for the children to discuss and build upon each other's ideas.

Rosen and Hoffman (2009) observed a first grade classroom where Mrs. Smith used virtual manipulatives to explore ways to represent and measure shapes. Mrs. Smith read a story to introduce the concept. Then, she had students engage in a computer program that included geoboards and pattern blocks. Finally, the students built models using concrete materials while drawing representations of their models. These activities allow students to achieve proficiency in first grade mathematics according to the National Council of Teaching Mathematics geometry standards. The data collected was based on teacher observation of the students as they interacted with the manipulatives during the learning process. This is a perfect example of how virtual manipulatives can be a useful instructional tool along with concrete materials.

Teachers need to allow student free exploration of manipulatives, have the materials packaged in accordance with the lesson, set clear learning goals, and model the use of materials (Joyner, 1990). Stein and Bovalino (2001) interviewed teachers who demonstrated competent teaching techniques when using manipulatives as tools. They shared three characteristics: they had extensive training in the use of manipulatives including workshops that let the teachers learn using concrete materials; they designed their own lessons and completed the task prior to presentation to anticipate student obstacles; and they spent ample time preparing the classroom and the manipulatives for the activity.

There are several factors that contribute to the lack of proper implementation of manipulatives in the classroom. Teachers complain that implementing the materials into their lessons is too time consuming. However, Suydam (1987) states that creating worksheets can take a comparable amount of time to create. Even when teachers do have time and access to concrete materials, many do not know how and when to utilize them (McBride & Lamb, 1986). The intentions of using these concrete materials can go astray when teachers expect students to master the skills too quickly or ask the students to complete the tasks step-by-step (Moch, 2001). The use of manipulatives can also become ineffective if teachers supply the materials to the students without instruction or guidance

(Stein & Bovalino, 2001). Teachers need to understand how to effectively use manipulatives as instructional teaching tools. According to Joyner (1990) teachers need management guidelines to effectively teacher using manipulatives. Jones, Uribe-Florez and Wilkins (2011) agreed that it is not whether teachers use the manipulatives, but rather how they used them. When used in ways that support students' control over learning, their competence, and relatedness to their teacher and their peers, teachers can help students develop an intrinsic motivation for learning.

Some teachers think of using manipulatives as chaotic rather than teaching tools and rely on written work to teach concepts (Joyner, 1990). These teachers do not believe manipulatives are essential to teaching and understanding (Green, Piel & Flowers, 2008). Historically, teachers viewed mathematics manipulatives as "fun." They would allow their students to use these tools at the end of the lesson, at the end of the week on Fridays, or at the end of the school year when district assessments were completed (Moyer, 2001). Researchers are trying to end these misconceptions and reverse the beliefs that manipulatives are unnecessary. Today, teachers are realizing that manipulatives are much more than that in the learning process and are willing to further their education to learn these strategies.

According to Puchner, Taylor, O'Donnell and Fick (2008) teachers must have a complete understanding of the mathematical content they are teaching. Green, Piel and Flowers (2008) found that by teaching pre-service teacher using manipulatives, their mathematical knowledge improved thus promoting recognition of the need to use them during instruction. Providing staff development in the proper use of manipulatives for teachers is critical for implementation to be successful. Teachers need to experience using hands-on materials for them to understand how effective they can be during instruction (Johnson, 1993). When students are guided by teachers who are knowledgeable about the use of manipulatives, their attitudes toward mathematics improves when using concrete materials according to Clements and McMillen (1996) and Sowell (1989).

Teacher and Student Outlooks

Teachers' attitudes towards mathematics are often transferred to their students according to Warkentin (1975). Most often teachers feel they do not have enough time to teach mathematics the way they know it should be taught thus leading to a poor attitude when teaching it. According to Krach (1998) the curriculums include teaching the concepts and operations but lack in allotting for the appropriate teaching time to effectively teach the development of concepts and operations. Instead, more time is allotted for rote memorization of rules and procedures. As a result, the students attempt to apply the rules when solving a problem but have no regard to understanding how the rules work. Teachers must utilize the appropriate concrete materials to form a "spirit-of-the-standards" approach to learning throughout their classroom environment that will transfer to students. This will assist in improving students' attitudes toward learning mathematics by becoming actively involved in their own learning. Cummings (1995) stated that teachers must use manipulatives to enhance students' excitement in the concepts of mathematics and problem solving strategies.

Teachers have concluded that students' success in learning mathematics depends upon their attitudes toward the subject and serious efforts should be made to promote such positive attitudes according to Farooq and Shah (2008). Using concrete materials enhances cooperative learning experiences, which then leads to active interest and involvement of the students involved in the task (Slavin, 1995). Virtual manipulatives improve students' willingness to take risks because they do not fear judgmental feedback on their errors as they may in a whole class activity according to Suh, Moyer and Heo (2005).

As mentioned earlier, some teachers view manipulatives as a "fun" activity. Glasser (1988) discusses that this is a positive outlook. He explains that students need fun just as much as they need belonging, power, and freedom. It is a good idea for students and teachers to view learning with manipulatives as a fun and active approach to learning concepts that have been viewed as frustrating if explored in a different capacity. This assists in encompassing a healthy learning experience and environment for the students. Suydam (1987) explains that students should be allowed to make "noise" while learning as this demonstrates that children are actively involved while sharing information with each other.

Ozgun-Koca and Edwards (2011) found that students preferred using manipulatives during instruction. Students found using them enjoyable and helpful when learning the concept. Deci, Koestner and Ryan (1999) discussed students' psychological needs to take control over their learning processes. Teachers need to strive to intrinsically motivate students' learning through the use of manipulatives because it leads to positive outcomes. Students who have more intrinsic motivation choose their own strategies and tools used for problem solving. Students who possess less intrinsic motivation simply follow a teacher's rules and procedures leading to poor understanding of abstract learning processes and lack of student achievement.

Student Achievement

According to the National Assessment of Educational Progress (National Center for Education Statistics, 2011), 60% of fourth-grade United States students scored less than proficient on mathematics assessments and only 10% of fourth-graders met advanced proficient goals in the international realm. President Barack Obama has launched an Educate to Innovate initiative that is designed to help students achieve high levels of mathematic proficiency by targeting contemporary instructional strategies (Carbonneau, Marley & Selig, 2013). Ball (1992) states that one important factor when improving mathematics education must be choosing the appropriate curriculum along with how and when it is implemented. The National Council of Teachers of Mathematics (NCTM, 2000) has advised that use of concrete manipulatives by teachers during instruction be integrated throughout the mathematics curriculum as well as students having access to the concrete manipulatives. Most students are only given the opportunity to use manipulatives during the lesson. This is a challenge because research shows that most students need extended periods of time manipulating physical models to develop fraction sense according to Cramer and Henry (2013). The minimum amount of time it takes for a student to grasp concepts through manipulative use depends upon the student, their intrinsic motivation, and their cognitive abilities.

Using manipulatives for reinforcement promotes higher scores when testing the retention of mathematical concepts (Suydam, 1986). Students who learn through the use of concrete materials at the elementary level outperform their peers at the secondary level stated Sowell (1989). Student achievement in ratio, proportion, and percent was found to be successful when the experience was extensive rather than occasional according to

Raphael and Wahlstrom (1989). Students need to use manipulatives repetitively in order to acquire transfer of a mathematical concept (Sowell, 1989). Parham (1983) and Suydam and Higgins (1977) agree that lessons taught using manipulatives produce higher student achievement in mathematics than lessons taught without using manipulatives.

Moyer-Packenham and Suh (2012) indicated in their study on student achievement that manipulatives, specifically virtual manipulatives, have different learning outcomes for students with different learning abilities. All of the students involved in this study demonstrated significant gains in achievement levels; however, each experience was unique due to the cognitive level of the student. For example, the high achieving group was able to complete tasks using mental math strategies and equivalency understanding while the low achieving group relied heavily on pictorial model to recognize those concepts. Thus, all groups demonstrated improvement when working with fractions using the virtual manipulatives.

McLeod and Armstrong (1982) found that students have extreme difficulty in the areas of fraction concepts whether they are found to have a learning disability or not. Reimer and Moyer (2005) found that all students demonstrated significant improvements in fraction understanding after using virtual manipulatives that included dynamic visuals of fraction amounts. They believe the use of the computer-based procedure was successful because it accommodated the pacing ability of all the students in the group; thus allowing for the higher level students to remain engaged and allowing the lower level students time to complete their given tasks. Burns and Hamm (2011) found that using concrete manipulatives verses virtual manipulatives to teach fractions to third graders resulted in the same overall improvement in student understanding.

Procedural knowledge is defined as knowing how to do something or recalling the algorithm to correctly formulate an answer. This can be memorized without any understanding of how the concept came to existence. On the other hand, conceptual knowledge which is defined as knowledge of interrelationships, offers students more flexibility in that they can invent a method to fill in a gap if they have forgotten a step or procedure (Anderson et al., 2001). Learning with manipulatives helps students build procedural fluency by increasing the level of engagement when using concrete materials in the future (Belenky & Nokes, 2009). Both procedural and conceptual knowledge are essential for success when solving a mathematics problem (Donovan & Bransford, 2005).

Cramer and Bezuk (1991) explain that using manipulatives to teach fractions is important because students need to conceptualize the concept. Teachers can supply the rules and students can memorize them. However, learning fractional concepts should focus on the interpretations involving two fractions and their product. Bohan and Shawaker (1994) suggest that students must progress through three stages for transfer to occur: concrete, bridging, and symbolic. First, students learn using and manipulating concrete materials hence the concrete stage. Next, students learn using both concrete materials and symbols representing the materials thus bridging the ideas together. Finally, the goal is for the students to problem solve using only symbols. Research has proven that the use of manipulatives is essential to learning mathematical concepts.

Not all research supports positive effects of using manipulatives. McNeil, Uttal, Jarvin and Sternberg (2009) examined student achievement when using concrete materials that were similar to real materials. They found that when students worked with real-world manipulatives (example coins and bills when calculating money was the task) many made errors; however, they were conceptual ones. Their conclusion was that presenting students with perceptually rich manipulatives had costs as well as benefits. Another concern is the lack of student support when transitioning from these real-world materials to the abstract mathematical concept (Clements & McMillen, 1996). Moyer (2001) agrees that manipulatives may hinder student success stating that manipulatives may be an extra step that may be too overwhelming for students.

While researching the topic, I found that their needs to be more studies conducted regarding specific representativeness of samples regarding diversity and gender as well as cognitive abilities. Most of the studies I researched were very broad and nonspecific in terms of the samples used in conducting the studies. Much of the data collected from the some studies of the studies included teacher observation of the students engaged in the use of the manipulatives.

Manipulatives are materials that allow students to concretize their knowledge by expressing concepts and performing problem-solving steps (Belenky & Nokes, 2009). Researchers examine many different types of manipulatives to pinpoint the best learning procedures for high student achievement. Although virtual manipulatives are an important tool in teaching mathematics, concrete materials are physical objects that can be linked to abstract ideas stated Burns and Hamm (2011). Researchers are still examining whether manipulatives really do improve student achievement. Butler et al. (2003) found that students who were instructed using the concrete-representational-abstract (CRA) procedure demonstrated better conceptual understanding than did those students who were instructed using the representational-abstract (RA) procedure. Suggestions for further study include teaching operations using the CRA procedure and compare the results with students being instructed using the traditional method of instruction.

Chapter 3

Methodology

Subjects

The subjects being used in this study consist of 18 fourth grade students assigned to the same teacher in a school that is identified by the state of New Jersey as a Title I school and a school in need of improvement. All of the students in this class live in a household identified as being low socioeconomic in status with 93% of the students receiving free and reduced lunch. The ethnic ratio of the school includes: 28 White, 88 Black, 677 Hispanic, 5 American Indian, 8 Asian, and 1 Multiracial.

The students range in age from nine to ten years old including thirteen females and five males; they have ethnicities of Caucasian, African American and Hispanic. Their academic abilities range from high achieving to specific learning disabled. There are two instructors including a regular education teacher and a special education teacher. The classroom setting is in-class resource with seven students receiving minimal modifications to the curriculum as per their Individualized Education Plan. Other students are pulled out of the classroom throughout the day for various additional services including English as a Second Language, Response to Intervention reading and math groups, speech services, and occupational therapy.

Variables

The students were administered a pretest and posttest to assess their knowledge base on fractional concepts and understanding before and after instruction. This assessment is a district wide assessment that was created by a collection of fourth grade teachers to include skills taught in Unit 7 of the Everyday Mathematics, McGraw Hill curriculum.

This assessment is representative of the assessments created and published by McGraw Hill to coincide with the Unit 7 curriculum and the Common Core Standards for Mathematics. It was reviewed and approved by the district's mathematics supervisor as well as the Vineland Board of Education. There were a total of twenty-seven questions; problems one through twenty-six were weighted four points each while problem twenty-seven weighted three points. There was an opportunity for a student to earn 111 points total.

The entire unit consists of twelve chapters. There are many key concepts and skills taught throughout this unit. They include: identify and name fractional parts of regions; identify fractions as equal parts of a whole or the ONE and solve problems involving fractional parts of regions and collections; identify equivalent fractions and mixed numbers; identify a triangle, hexagon, trapezoid, and rhombus; find fractions and mixed numbers on number lines; identify the whole or ONE when given the "fraction-of;" use an equal-sharing division strategy; add fractions with like and unlike denominators; use basic probability terms to describe and compare the likelihood of an event; express the probability of an event as a fraction; find fractional parts of polygonal regions; model fraction addition and subtraction with pattern blocks; represent fractions with pattern blocks; use patterns in a table to find equivalent fractions; develop and use a rule for generating equivalent fractions; represent a shaded region as a fraction and a decimal; rename fractions with 10 and 100 in the denominator as decimals; use fraction notation and equal sharing to solve division problems; compare fractions and order fractions as well as explain their strategies; given a fractional part of a region or collection, name the ONE; use equivalent fractions to design spinners; rename fractions as percents; and use fractions and percents to predict the outcomes of an experiment. These skills overlap and intertwine within the chapters. Some skills require an entire 75 minute math period while others require two 75 minute periods. These time periods include but do not always necessitate math small groups.

Procedures

The students were divided into two groups of nine. Each group included various ethnicities, genders, and learning abilities. Each student was given a number for anonymous identification of student work and to monitor progress data. Each student was administered a pretest to assess their knowledge of fractional concepts. The data was recorded on a spreadsheet. The groups were instructed in separate rooms during the scheduled math block.

The students in group one were instructed on the various concepts and understanding of fractions including addition and subtraction following the pretest. This took place for approximately four weeks. They watched manipulatives being modeled by the teachers, interacted with each other using manipulatives, work independently using manipulatives, and participated in computer programs using manipulatives throughout the four week learning process. The two teachers in the room took turns teaching each group separately. This ensured that teaching style would not be a limitation in the study.

The students in group two were also instructed on the various concepts and understanding of fractions including addition and subtraction following the pretest which took place for four weeks. However, they only watched the teachers model the skill using manipulatives. The students did not directly use the hands-on manipulatives during the learning process.

Both groups completed homework from a workbook provided by the Everyday Mathematics curriculum. Each lesson includes a Study Link page that reviews the lesson taught in class on each day except Fridays. The students were asked to complete the corresponding page to the day's lesson each night as independently as possible. The homework was reviewed by the teachers each day. The teachers held conferences with students who had difficulty with specific concepts.

Group One: Manipulative Use

During the first week, the special education teacher taught group one using manipulatives. She used direct instruction while writing on the board, used an overhead projector to illustrate the concepts, and allowed each child to work with their own set of 2 dimensional pattern block shapes. The first chapter (7.1) was taught for one 75 minute whole group lesson. It included having the students divide the ONE (a hexagon) into equal parts using other shapes including trapezoids, rhombi, and triangles. It included having students place fractions and mixed numbers on number lines. The students listened to the lessons while following along in their workbooks. The teacher used pattern blocks on the overhead projector to illustrate the concepts while the students used the pattern blocks at their desks and wrote the answers in their workbooks. They completed classwork following the instruction independently.

The next chapter (7.2) was taught for one 75 minute whole group lesson, one 55 minute whole group lesson, and one 20 minute small group period. It included having the students finding fractional sets of a whole. The students were each given approximately 20 counters to use during the lesson. The teacher modeled using the counters followed by the children using them as they continued through the workbook pages together as a class. They were taught the formula "divide under, multiply over" to solve these problems also. For example, if the problem was one-third of twelve, the student would divide 12 by the denominator 3 to equal 4, then multiply 4 the numerator 1 to equal 4.

Problem solving exercises were completed in the next lesson. The students were given a packet of word problems which were compiled by teachers in the district along with counters to use at their desks. An example of a problem included: Michael had 20 baseball cards. He gave one-fifth of them to his friend Alena, and two-fifths to his brother Dean. How many baseball cards did he give to Alena? Problem one was completed by the teacher as the students observed her strategies. Problem two was completed independently and reviewed as a class. Problem three was completed in pairs and reviewed as a class.

The following chapter (7.3) was taught during one 55 minute whole group lesson and one 20 minute small group time. It included creating fractions when finding probabilities to events when all possible outcomes are equally likely. The class was divided into pairs and each pair was given a deck of playing cards. The students watched the teacher model the concept using a full deck of playing cards. Together, they completed the first problem on the workbook page which included creating fractions based on probability when drawing from a deck of cards. Each pair completed the workbook pages together using the deck of playing cards. During the 20 minute small group time, the students cut out fraction cards that will be used for future lessons while filling in the missing numerators or denominators. Some students were pulled to work with the teacher for extra support.

On the final day of the week, the students began the next chapter (7.4) which included learning how to find fractional parts of polygonal regions during one 45 minute lesson and one 30 minute small group time. Each student was given a set of 2 dimensional shapes to use at their desks. The teacher modeled covering hexagons with 2-dimentional shapes including trapezoids, rhombi, and triangles on the overhead. The students followed along and they completed workbook pages as a class while using their 2 dimensional shapes. During small group time, the students took turns rotating between math centers that included fraction flash cards, fraction BINGO, and interactive fraction computer games.

During the second week, the regular education teacher taught the lessons for group one using manipulatives. On the first two days of the week, the pattern-block fraction chapter (7.4) was continued for one 75 minute whole group lesson. Again, each student was given a set of 2 dimensional shapes. The students watched as the teacher modeled using shapes to create a picture using the 2 dimensional shapes and then writing the parts as fractions. She also modeled creating shapes by tracing the shapes and by using a straight edge for when they do not have the shapes. The students worked in pairs to complete an alternative assessment for 30 minutes. The alternative assessment included creating their own picture or object using the 2 dimensional shapes and recording the parts as a fraction. The students used a straight edge to create the shapes also.

The next chapter (7.5) took place for the rest of the week. It included adding and subtracting fractions with like and unlike denominators. During the first 75 minute whole group lesson, the teacher explained how to find equivalent fractions as the students referred to their fraction cards they previously cut out. She modeled using pieces of a fraction bar on the overhead projector. She modeled using a multiplication chart while multiplying the numerator and denominator by the same number to equal the equivalent fraction. She gave the students opportunities to try given problems. They worked in pairs to solve given problems. Then, as a class, they completed a workbook page.

For the following day's chapter (7.7) which included 55 minutes of whole group lesson and 20 minutes of small group time, the students began by completing a worksheet independently. Each student had a set of fraction bar pieces and their fraction cards. They were required to find equivalent fractions using either the fraction bar pieces or their fraction cards. The worksheet was reviewed by the whole group followed by a short question and answer session. The teacher modeled adding and subtracting fractions with like denominators while explaining that the denominator must remain the same; only the numerator is added together. They completed workbook pages with a partner that was using the same type of fraction manipulative as them. During the small group time, some students played the Fraction of a Pizza board game while others worked with the teacher for additional support.

During the last lesson of the week, the students observed as the teacher explained and modeled how to add and subtract fractions with unlike denominators while referring to the previous two chapters (7.5 and 7.7) to link concepts. This was competed during a 75 minute whole group lesson. Each student was given a set of fraction bars. The teacher used fraction bars to illustrate concepts on the overhead projector as she completed the first two problems in the workbook. They worked independently to complete workbook pages using the fraction bars. They reviewed the problems as a whole class.

The special education teacher instructed group one using manipulatives during week three. The first chapter (7.6) reviewed equivalent fractions by finding many names for fractions. This took place for 55 minutes in a whole group setting and 20 minutes for small group time. The students completed workbook pages as a class. They included having the students color missing squares from given rectangles equally divided into sections and filling in the missing denominator from the given fractions. The completed workbook pages in pairs where they listed at least three equivalent fractions equal to the given fractions. They used a multiplication chart to complete this activity. During small group time, some students worked with interactive fraction computer games and other students worked with the teacher for additional support.

The next chapter (7.8) was completed during one 75 minute whole group lesson, one 55 minute whole group lesson, one 45 minute whole group lesson, one 30 minute small group time, and one 20 minute small group time over a three day period. This included providing experience with renaming fractions as decimals and decimals as fractions. They developed an understanding of the relationship between fractions and division. For the first lesson, each student was given a set of base-ten blocks. The students observed the teacher modeling how to convert fractions into decimals with the denominators of 10 and 100. She used base-ten grid blocks to illustrate the concept on the overhead projector. The class completed the corresponding workbook pages together as a whole group while using the base-ten blocks.

The following lesson began with 20 minutes of small group time. Each student was given a set of base-ten blocks. The students exchanged papers, graded each other's homework, and helped each other correct mistakes. During the 55 minute whole group lesson, the teacher retaught the concept of converting fractions with denominators of 10 and 100 to decimals using base-ten blocks on the overhead projector. The students used their base-ten blocks to find answers to given problems. They shared their answers on the board.

The next lesson began with a 45 minute whole group lesson. The teacher explained and modeling the relationship between fractions and decimals using a calculator. The students were given calculators to better understand the relationship between fractions and decimals as they converted given fractions to decimals. Students worked with interactive fraction computer games and other students worked with the teacher for additional support during the 30 minute small group time.

The regular education teacher completed teaching the lessons in the unit with group one without using manipulatives during the fourth week. The first chapter (7.9) began with one 75 minute whole group lesson. It included comparing fractions and provided experience ordering sets of fractions. Each student was given a set of fraction bars. The teacher explained the concept using fraction bars on the overhead projector while the students observed and copied her strategy with their fraction bars. Together as a class, they completed two workbook pages using the manipulatives.

The next chapter (7.10) guided students as they found the ONE or the whole for given fractions. This was taught during one 75 minute whole group lesson, one 45 minute lesson, and one 30 minute small group lesson. Each student was given a set of 2 dimensional pattern block shapes and approximately 20 counters. The teacher used pattern blocks to illustrate how and why the given pattern blocks represented the ONE. For example, one triangle represents one-sixth of the ONE because it takes six triangles to cover a whole hexagon/ONE. Counters were also used to represent the ONE of a given event. They completed the workbook pages together as a class using the appropriate manipulative (pattern blocks or counters).

During the following lesson, the students worked in pairs. Each pair was given a set of 2 dimensional pattern block shapes. The teacher reviewed the parts of the ONE for each pattern block. She modeled examples of the ONE using counters. The students collaborated to solve given problems including "given the fraction how many counters would that represent from the ONE." Students were required to draw their problem representing the counters and grouping of the fraction. For example, if the fraction was one-half, they would draw 10 counters to represent the ONE and circle 5 of them to represent the fraction. During small group time, the students took turns rotating between math centers that included fraction flash cards, and fraction BINGO.

The final chapter (7.11) of the unit was taught during one 45 minute whole group lesson and one 30 minute small group time. The concepts were to find the probability when using a spinner and write it as a fraction. The teacher modeled using a spinner using a large model on the board. She listed scenarios, students took turns spinning, and writing the fractions on the board. They completed the workbook page together as a whole class. During the 30 minute small group time, the students took turns playing the Fraction of a Pizza and working on the computer. The computer game consisted of probability when using a spinner. A test review including examples of all the skills taught in Unit 7 was given for homework.

Group Two: No Manipulative Use

During the first week, the regular education teacher taught group two without using manipulatives. She used direct instruction while writing on the board and using an overhead projector to illustrate the concepts. The first chapter (7.1) was taught for one 75 minute whole group lesson. It included having the students divide the ONE (a hexagon) into equal parts using other shapes including trapezoids, rhombi, and triangles. It included having students place fractions and mixed numbers on number lines. The students listened to the lessons while following along in their workbooks. The teacher used pattern blocks on the overhead projector to illustrate the concepts. They completed classwork following the instruction independently.

The next chapter (7.2) was taught for two 75 minute whole group lessons. It included having the students finding fractional sets of a whole. During the first day, the students listened to the lessons while following along in their workbooks. They were taught the formula “divide under, multiply over”. They observed the teacher as she further explained the concept by using counters on the overhead projector. They completed the workbook pages together as a class.

Problem solving exercises were completed during the next day of the lesson. The students were given a packet of word problems which were compiled by teachers in the district. Problem one was completed by the teacher as the students observed her strategies. Problem two was completed independently and reviewed as a class. Problem three was completed in pairs and reviewed as a class.

The following chapter (7.3) was taught during one 55 minute whole group lesson and one 20 minute small group time. It included creating fractions when finding probabilities to events when all possible outcomes are equally likely. The students watched the teacher model the concept using a full deck of playing cards. Together, they completed the workbook pages which included creating fractions based on probability when drawing from a deck of cards. The students closed the lesson by completing a workbook page independently while some students were pulled to work with the teacher for extra support.

On the final day of the week, the students began the next chapter (7.4) which included learning how to find fractional parts of polygonal regions during one 75 minute whole group lesson. The teacher modeled covering hexagons with 2-dimentional shapes including trapezoids, rhombi, and triangles on the overhead. The students followed along and they completed workbook pages as a class.

During the second week, the special education teacher taught the lessons for group two without using manipulatives. On the first two days of the week, the pattern-block fraction chapter (7.4) was continued for one 75 minute whole group lesson. The students watched as the teacher modeled using shapes to create a picture using the 2 dimensional shapes and then writing the parts as fractions. She also modeled creating shapes using a straight edge. The students worked in pairs to complete an alternative assessment for 30 minutes. The alternative assessment included creating their own picture or object drawing the 2 dimensional shapes and recording the parts as a fraction. The students used a straight edge to create the shapes.

The next chapter (7.5) took place for the rest of the week. It included adding and subtracting fractions with like and unlike denominators. During the first 75 minute whole group lesson, the teacher explained how to find equivalent fractions. She modeled using a multiplication chart while multiplying the numerator and denominator by the same number to equal the equivalent fraction. She also illustrated the concept by showing fraction bars on the overhead. She gave the students opportunities to try given problems. They worked in pairs to solve given problems. Then, as a class, they completed a workbook page.

For the following day's chapter (7.7) which included 55 minutes of whole group lesson and 20 minutes of small group time, the students began by completing a worksheet independently. They were required to find equivalent fractions. The worksheet was reviewed by the whole group followed by a short question and answer session. The teacher modeled adding and subtracting fractions with like denominators while explaining that the denominator must remain the same; only the numerator is added. They completed workbook pages as a class which used pattern blocks to illustrate adding and subtracting the fractions. During the small group time, some students completed a worksheet while others worked with the teacher.

During the last lesson of the week, the students observed as the teacher explained and modeled how to add and subtract fractions with unlike denominators while referring to the previous two chapters (7.5 and 7.7) to link concepts. This was competed during a 75 minute whole group lesson. Again, she used pattern blocks and fraction bars to illustrate concepts on the overhead projector. They worked independently to complete workbook pages. They reviewed them as a whole class.

The regular education teacher instructed group two without using manipulatives during week three. The first chapter (7.6) reviewed equivalent fractions by finding many names for fractions. This lesson took place for 75 minutes and was taught in a whole group setting. The students completed workbook pages as a class. They included having the students color missing squares from given rectangles equally divided into sections and filling in the missing denominator from the given fractions. The completed workbook pages in pairs where they listed at least three equivalent fractions equal to the given fractions. They used a multiplication chart to complete this activity.

The next chapter (7.8) was completed during one 75 minute whole group lesson, one 55 minute whole group lesson, one 45 minute whole group lesson, one 30 minute small group time, and one 20 minute small group time over a three day period. This included providing experience with renaming fractions as decimals and decimals as fractions. They developed an understanding of the relationship between fractions and division. For the first lesson, the students observed the teacher modeling converting fractions into decimals with the denominators of 10 and 100. She used base-ten grid blocks to illustrate the concept on the overhead projector. The class completed the corresponding workbook page together as a whole group.

The following lesson began with 20 minutes of small group time. The students exchanged papers, graded each other's homework, and helped each other correct mistakes. During the 55 minute whole group lesson, the teacher retaught the concept of converting fractions with denominators of 10 and 100 to decimals using base-ten blocks on the overhead projector. Students took turns answering given problems on the board.

The next lesson began with a 45 minute whole group lesson. The teacher explained and modeled the relationship between fractions and decimals using a calculator. The students were given calculators to better understand the relationship between fractions and decimals as they converted given fractions to decimals. Students worked in math centers on concepts other than fractions during the 30 minute small group time.

The special education teacher completed teaching the lessons in the unit with group two without using manipulatives during the fourth week. The first chapter (7.9) began with one 75 minute whole group lesson. It included comparing fractions and provided experience ordering sets of fractions. The teacher explained the concept using fraction bars on the overhead projector while the students observed. Together as a whole class, they completed two workbook pages. They completed another workbook page independently.

The next chapter (7.10) guided students as they found the ONE or the whole for given fractions and was taught during a 75 minute whole group lesson. The teacher used pattern blocks to illustrate how and why the given 2 dimensional shapes represented the ONE. For example, one triangle represents one-sixth of the ONE because it takes six triangles to cover a whole hexagon/ONE. Counters were also used to represent the ONE of a given event. They completed the workbook pages together as a class. The pace of this lesson was very slow as the students had difficulty understanding this concept.

The following lesson was during one 45 minute whole group lesson and one 30 minute small group time. During this lesson, the teacher reviewed the parts of the ONE for each pattern block. She modeled examples of the ONE using counters. She placed given problems using counters on the overhead projector and students solved them independently. For the 30 minute small group time, students collaborated in groups of three to solve given problems including "given the fraction how many counters would that represent from the ONE." Students were required to draw the problem representing the counters and grouping of the fraction. For example, if the fraction was one-half, they would draw 10 counters to represent the ONE and circle 5 of them to represent the fraction.

The final chapter (7.11) of the unit was taught during one 55 minute whole group lesson and one 20 minute small group time. The concepts were to find the probability when using a spinner and write it as a fraction. The teacher modeled using a spinner using a large model on the board. She listed scenarios, students took turns spinning, and writing the fractions on the board. They completed the workbook page together as a whole class. During the 20 minute small group time, the students completed the remaining workbook pages in pairs. A test review including examples of all the skills taught in Unit 7 was given for homework.

Statistical Analysis

On the last day of the week, the lesson began with a review of the homework. A brief question and answer session took place. The test was administered by the regular education teacher. The students with Individual Education Plans were taken to a separate room and the test was administered by the special education teacher. This is the same assessment that was administered as the pretest. The data from the pretest and the posttest was analyzed using a t-test to measure the amount of growth achieved by both groups.

Chapter 4

Results

The pre and post assessments given to the students included skills taught in McGraw Hill's Everyday Mathematics curriculum regarding fractions. The first group of students that used manipulatives that observed manipulatives being modeled by the teachers, interacted with each other using manipulatives, work independently using manipulatives, and participated in computer programs using manipulatives during the learning process demonstrated more academic growth than the second group of students that did not use manipulatives during the learning process. A repeated measures t-test determined that growth varies significantly according to manipulative use, t (16) = -5.721, p = .000.

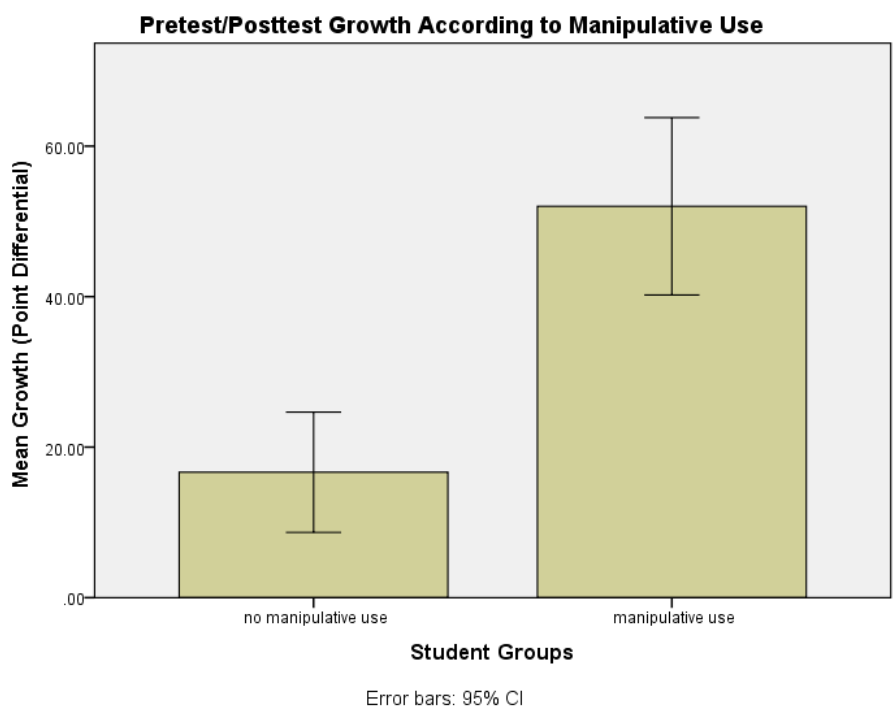

Figure 1. Comparing growth of correlations according to manipulative use.
Note. ***Finding is significant at p < .000.

The students that did not use manipulatives during the learning process demonstrated a small amount of growth (M = 16.17, SD = 10.4). However, the students that used manipulatives throughout the learning process showed a significant amount of growth (M = 52, SD = 15.33). As discussed in my research, students who use concrete hands-on manipulatives while learning conceptualize and internalize concepts.

These results are an important indicator that if educators use manipulatives when teaching fractions, then students would successfully internalize fractional concepts thus demonstrating significant student growth. They also prove that the use of manipulatives when teaching fractions is more effective than teaching fractions using the paper pencil style of teaching the concepts.

Chapter 5

Discussion

Conclusions Regarding Effectiveness of Manipulative Use

The presented findings reveal significant information about the focus on the effectiveness of using manipulatives when teaching fractions to elementary school students. The teaching style in the United States has been rapidly moving from old school worksheets and direct instruction to a more hands-on approach. It is important to research these new teaching methods to ensure educators are moving in the right direction to promote student growth and achievement.

The students who participated in the study included 18 fourth grade students ranging in age from nine to ten years old. First, all the participants were given a pre assessment to assess their knowledge of fraction concepts and skills. Next, they were divided into two groups with ability level, ethnicity, and gender evenly distributed. Each group was instructed by the regular education teacher and the special education teacher at different times. The two teachers rotated teaching each group weekly for a four week period. The curriculum used was provided by the Vineland Public Schools District which they purchased from McGraw Hill Publishing, Everyday Mathematics. Group one was instructed using manipulatives throughout the learning process. Group two was instructed without using manipulatives but rather a direct instruction approach.

As the hypothesis indicated if educators use manipulatives when teaching fractions, then students would successfully internalize fractional concepts thus demonstrating significant student growth. The use of manipulatives when teaching fractions is more effective than teaching fractions using the paper pencil style of teaching the concepts. The results of the study indicated that the students that were instructed using the direct instruction approach and teacher modeling did show growth. However, the students that were instructed direct instruction and teacher modeling along with manipulatives during the learning process demonstrated a significant ($p = .000$) amount of student growth as compared to the non-manipulative group.

The students who were being instructed using only direct instruction and teacher modeling were not engaged in their learning process. These students were distractible, were not willing to participate, and did not go beyond the means of what was expected of them. This was evident when completing the alternative assessment. This assignment was submitted either not completed or completed in a very sloppy manner. During instructional time these students were observed playing with pencils, watching the clock, staring into space/daydreaming, and moving ahead through the workbook. They also asked to use the restroom frequently during this scheduled time block. A minimal amount of student growth was present was these students were assessed ($M = 16.17$, $SD = 10.4$). Cramer and Bezuk (1991) explain that students need to use manipulatives when learning fractions. Although teachers can provide the rules for students to memorize them, they need to conceptualize the concepts.

The students who were being instructed using manipulatives were excited and engaged during the learning process. They were willing to share ideas and explore concepts. They asked and answered higher order thinking questions including creating, analyzing, understanding, and applying additional concepts. All students were attentive to the task at hand and were willing to participate. These students showed a significant amount of growth when assessed ($M = 52$, $SD = 15.33$).

As research has indicated, using concrete materials along with metacognitive prompts by teachers is critical to internalizing complex cognitive problem solving skills. The use of manipulatives grounds new information in prior knowledge and enables students to abstract the critical features through reflection thus leading to higher student achievement (Belenky & Nokes, 2009).

The students used 2 dimensional pattern block shapes, fraction bars, board games, and interactive computer programs/games as their manipulatives. Teachers need to allow student free exploration of manipulatives, have the materials packaged in accordance with the lesson, set clear learning goals, and model the use of materials (Joyner, 1990). During this research study, all the manipulatives were packaged per student and the corresponding manipulatives were distributed per lesson. The students were allowed to manipulate them as the teacher was modeling how and when to use them. The manipulatives were used many times throughout the four week period.

During the lessons where pattern blocks and fraction bars were manipulated by the students, they were very interactive with the lesson objectives. The students frequently made comments throughout the lesson such as, "This is fun! or Oh, I get it!" They were very willing to help a classmate by illustrating the concept using the pattern blocks. I feel students often times have difficulty verbalizing ideas. As I observed, using the concrete objects allowed for the students to communicate their thoughts and ideas. While playing the Fraction of a Pizza game, I observed the students laughing and teaching each other the skills. Although their participation with the computer programs was not monitored or assessed, they were engaged in those lessons.

Limitations

There are a few limitations with this study that were a result of the location as well as the population of the participants. Because the study was conducted with minors in a public school setting, there were regulations of the district's mathematics curriculum with regards to material and pacing. Slower pacing along with the use of additional materials with regards to manipulatives, would allow for students to better retain the concepts. Another limitation is that this study took place during a time period of only four weeks. Based on the amount of the material and the depth of the concepts, this is not a significant amount of time to allow the students to fully conceptualize all the skills needed to achieve advanced proficiency on the post assessment. More amounts of teaching time with regards to more scheduled mathematics blocks, would allow for greater student achievement. Another limitation to this study is that seven of the participants have specific learning disabilities. This hinders their ability to retain mathematical concepts without repetition over a longer period of time. Another limitation is that there are only 18 participants which is not a large subject pool. An additional limitation is that the assessments are paper pencil based and do not allow for oral responses or virtual responses which may be necessary for some students to successfully demonstrate their mastery of the skill.

Further Direction

This research study lends itself to further research. The materials used in this study were abstract materials. Future research should address whether the use of real world materials would result in superior learning as compared to materials that are artificial and abstract. Another question that needs to be further researched would be in regards to matching manipulatives to the appropriate concept. Are all manipulatives universal to all fractional concepts? There are many different manipulatives that can be used when teaching fractions. To enhance student achievement goals, educators need to understand which manipulatives to use when teaching specific fraction skills.

An additional research study could be done on how long the skills are actually retained for in regards to long term memory verses short term memory. This study assessed student knowledge immediately after four weeks of instruction. Would these students show the same amount of growth when assessed two months from now? Another post assessment would need to be given two months from now to further this study.

The goal of teachers and administrators throughout the country is for our educational system to ensure student growth and achievement. Because all students learn differently, it is important to provide a multidisciplinary approach to teaching. Using concrete, hands-on manipulatives during the learning process can promote student growth. Parham (1983) and Suydam and Higgins (1977) agree that lessons taught using manipulatives produce higher student achievement in mathematics than lessons taught without using manipulatives.

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